BOSTES
BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

MATHEMATICS EXTENSION 2
STAGE 6

DRAFT SYLLABUS FOR CONSULTATION

20 JULY – 31 AUGUST 2016

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Published by:
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D2016/49787

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THE BOSTES SYLLABUS DEVELOPMENT PROCESS

BOSTES began its syllabus development process for Stage 6 English, Mathematics, Science and History in 2014. This followed state and territory Education Ministers' endorsement of senior secondary Australian curriculum.

The development of the Stage 6 syllabuses involved expert writers and opportunities for consultation with teachers and other interest groups across NSW in order to receive the highest-quality advice across the education community.

A number of key matters at consultations were raised, including the need for the curriculum to cater for the diversity of learners, the broad range of students undertaking Stage 6 study in NSW, development of skills and capabilities for the future, school-based assessment and providing opportunities for assessing and reporting student achievement relevant for post-school pathways.

There was broad support that changes to curriculum and assessment would contribute to the reduction of student stress. BOSTES will continue to use NSW credentialling processes aligned with Stage 6 assessment and HSC examination structures.

A summary of the BOSTES syllabus development process is available at http://www.boardofstudies.nsw.edu.au/syllabuses/syllabus-development.

ASSISTING RESPONDENTS

The following icons are used to assist respondents:

for your information	This icon indicates general information that assists in reading or understanding the information contained in the document. Text introduced by this icon will not appear in the final syllabus.
consult	This icon indicates material on which responses and views are sought through consultation.

CONSULTATION

The *Mathematics Extension 2 Stage 6 Draft Syllabus* is accompanied by an online consultation survey on the BOSTES website. The purpose of the survey is to obtain detailed comments from individuals and systems/organisations on the syllabus. Please comment on both the strengths and the weaknesses of the draft syllabus. Feedback will be considered when the draft syllabus is revised.

The consultation period is from 20 July to 31 August 2016.

Written responses may be forwarded to:

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INTRODUCTION

STAGE 6 CURRICULUM

Board of Studies, Teaching and Educational Standards NSW (BOSTES) Stage 6 syllabuses have been developed to provide students with opportunities to further develop skills which will assist in the next stage of their lives, whether that is academic study, vocational education or employment.
The purpose of the Higher School Certificate program of study is to:

- provide a curriculum structure which encourages students to complete secondary education
- foster the intellectual, social and moral development of students, in particular developing their:
  - knowledge, skills, understanding, values and attitudes in the fields of study they choose
  - capacity to manage their own learning
  - desire to continue learning in formal or informal settings after school
  - capacity to work together with others
  - respect for the cultural diversity of Australian society
- provide a flexible structure within which students can prepare for:
  - further education and training
  - employment
  - full and active participation as citizens
- provide formal assessment and certification of students' achievements
- provide a context within which schools also have the opportunity to foster students' physical and spiritual development.

The Stage 6 syllabuses reflect the principles of the BOSTES *K–10 Curriculum Framework* and *Statement of Equity Principles*, and the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008). The syllabuses build on the continuum of learning developed in the K–10 syllabuses.

The Stage 6 syllabuses provide a set of broad learning outcomes that summarise the knowledge, understanding, skills, values and attitudes essential for students to succeed in and beyond their schooling. In particular, the literacy and numeracy skills needed for future study, employment and life are provided in Stage 6 syllabuses in alignment with the *Australian Core Skills Framework (ACSF)*.

The syllabuses have considered agreed Australian curriculum content and included content that clarifies the scope and depth of learning in each subject.

Stage 6 syllabuses support a standards-referenced approach to assessment by detailing the essential knowledge, understanding, skills, values and attitudes students will develop and outlining clear standards of what students are expected to know and be able to do. In accordance with the *Statement of Equity Principles*, Stage 6 syllabuses take into account the diverse needs of all students. The syllabuses provide structures and processes by which teachers can provide continuity of study for all students.

DIVERSITY OF LEARNERS

NSW Stage 6 syllabuses are inclusive of the learning needs of all students. Syllabuses accommodate teaching approaches that support student diversity including Students with special education needs, Gifted and talented students and Students learning English as an additional language or dialect (EAL/D).

STUDENTS WITH SPECIAL EDUCATION NEEDS

All students are entitled to participate in and progress through the curriculum. Schools are required to provide additional support or adjustments to teaching, learning and assessment activities for some students. Adjustments are measures or actions taken in relation to teaching, learning and assessment that enable a student to access syllabus outcomes and content and demonstrate achievement of outcomes.

Students with special education needs can access the Stage 6 outcomes and content in a range of ways. Students may engage with:

- syllabus outcomes and content with adjustments to teaching, learning and/or assessment activities
- selected outcomes and content appropriate to their learning needs
- selected Stage 6 Life Skills outcomes and content appropriate to their learning needs.

Decisions regarding adjustments should be made in the context of collaborative curriculum planning with the student, parent/carer and other significant individuals to ensure that syllabus outcomes and content reflect the learning needs and priorities of individual students.

Further information can be found in support materials for:

- Mathematics
- Special education needs
- Life Skills.

GIFTED AND TALENTED STUDENTS

Gifted students have specific learning needs that may require adjustments to the pace, level and content of the curriculum. Differentiated educational opportunities assist in meeting the needs of gifted students.

Generally, gifted students demonstrate the following characteristics:

- the capacity to learn at faster rates
- the capacity to find and solve problems
- the capacity to make connections and manipulate abstract ideas.

There are different kinds and levels of giftedness. Gifted and talented students may also possess learning difficulties and/or disabilities that should be addressed when planning appropriate teaching, learning and assessment activities.

Curriculum strategies for gifted and talented students may include:

- differentiation: modifying the pace, level and content of teaching, learning and assessment activities
- acceleration: promoting a student to a level of study beyond their age group
- curriculum compacting: assessing a student’s current level of learning and addressing aspects of the curriculum that have not yet been mastered.

School decisions about appropriate strategies are generally collaborative and involve teachers, parents and students with reference to documents and advice available from BOSTES and the education sectors.

Gifted and talented students may also benefit from individual planning to determine the curriculum options, as well as teaching, learning and assessment strategies, most suited to their needs and abilities.

STUDENTS LEARNING ENGLISH AS AN ADDITIONAL LANGUAGE OR DIALECT (EAL/D)

Many students in Australian schools are learning English as an additional language or dialect (EAL/D). EAL/D students are those whose first language is a language or dialect other than Standard Australian English and who require additional support to assist them to develop English language proficiency.

EAL/D students come from diverse backgrounds and may include:

- overseas and Australian-born students whose first language is a language other than English, including creoles and related varieties
- Aboriginal and Torres Strait Islander students whose first language is Aboriginal English, including Kriol and related varieties.

EAL/D students enter Australian schools at different ages and stages of schooling and at different stages of English language learning. They have diverse talents and capabilities and a range of prior learning experiences and levels of literacy in their first language and in English. EAL/D students represent a significant and growing percentage of learners in NSW schools. For some, school is the only place they use English.

EAL/D students are simultaneously learning a new language and the knowledge, understanding and skills of the Mathematics Extension 2 Stage 6 syllabus through that new language. They require additional time and support, along with informed teaching that explicitly addresses their language needs, and assessments that take into account their developing language proficiency.

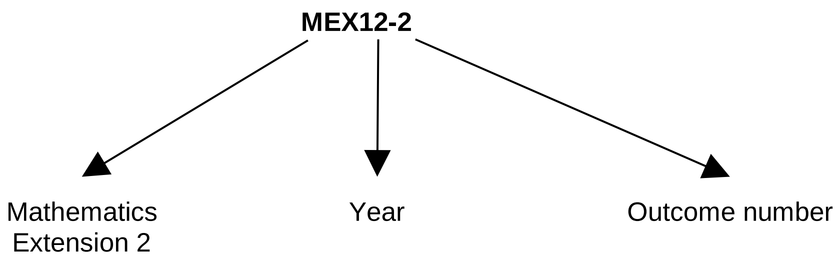
MATHEMATICS EXTENSION 2 KEY

The following codes and icons are used in the *Mathematics Extension 2 Stage 6 Draft Syllabus*.

OUTCOME CODING

Syllabus outcomes have been coded in a consistent way. The code identifies the subject, Year and outcome number.

In the *Mathematics Extension 2 Stage 6 Draft Syllabus*, outcome codes indicate the subject, Year and outcome number. For example:

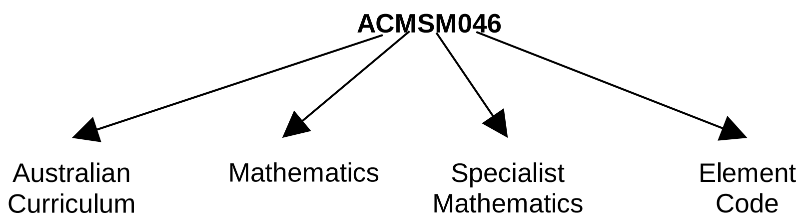


Outcome code	Interpretation
MEX12-4	Mathematics Extension 2, Year 12 – Outcome number 4

CODING OF AUSTRALIAN CURRICULUM CONTENT

Australian curriculum content descriptions included in the syllabus are identified by an Australian curriculum code which appears in brackets at the end of each content description, for example:

Prove and apply the Pythagorean identities (ACMSM046).



Where a number of content descriptions are jointly represented, all description codes are included, eg (ACMMM001, ACMMM002, ACMSM003).

CODING OF LEARNING OPPORTUNITIES

The syllabus provides opportunities for modelling applications and exploratory work. These should enable candidates to make connections and appreciate the use of mathematics and appropriate digital technology.

M	This identifies an opportunity for explicit **m**odelling applications or investigations that may or may not involve real-life applications or cross-strand integration.
E	This identifies opportunities for extended **e**xploratory work. Such opportunities allow students to investigate in a rich way the evolution of mathematics or mathematics in practice. Opportunities such as this could form the basis of an internal, non-examination based assessment and are outside the scope of the HSC examination.

As these opportunities are both an integral part of each strand and merge strands together, they are identified by the letter at the end of the relevant content description.

For example: Identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042) **M**

LEARNING ACROSS THE CURRICULUM ICONS

Learning across the curriculum content, including cross-curriculum priorities, general capabilities and other areas identified as important learning for all students, is incorporated and identified by icons in the *Mathematics Extension 2 Stage 6 Draft Syllabus*.

Cross-curriculum priorities	
	Aboriginal and Torres Strait Islander histories and cultures
	Asia and Australia's engagement with Asia
	Sustainability
General capabilities	
	Critical and creative thinking
	Ethical understanding
	Information and communication technology capability
	Intercultural understanding
	Literacy
	Numeracy
	Personal and social capability
Other learning across the curriculum areas	
	Civics and citizenship
	Difference and diversity
	Work and enterprise

RATIONALE

for your information

The rationale describes the distinctive nature of the subject and outlines its relationship to the contemporary world and current practice. It explains the place and purpose of the subject in the curriculum, including:

- why the subject exists
- the theoretical underpinnings
- what makes the subject distinctive
- why students would study the subject
- how it contributes to the purpose of the Stage 6 curriculum
- how it prepares students for post-school pathways.

consult

Mathematics is the study of order, relation, pattern, uncertainty and generality and is underpinned by observation, logical reasoning and deduction. From its origin in counting and measuring, its development throughout history has been catalysed by its utility in explaining real-world phenomena and its inherent beauty. It has evolved in highly sophisticated ways to become the language now used to describe many aspects of the modern world.

Mathematics is an interconnected subject that involves understanding and reasoning about concepts and the relationships between those concepts. It provides a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The Mathematics Stage 6 syllabuses are designed to offer opportunities for students to think mathematically. Mathematical thinking is supported by an atmosphere of questioning, communicating, reasoning and reflecting and is engendered by opportunities to generalise, challenge, find connections and to think critically and creatively.

All Mathematics Stage 6 syllabuses provide opportunities to develop students' 21st–century knowledge, skills, understanding, values and attitudes. As part of this, in all courses students are encouraged to learn with the use of appropriate technology and make appropriate choices when selecting technologies as a support for mathematical activity.

Mathematics Extension 2 provides students with the opportunity to develop strong mathematical manipulative skills and a deep understanding of the fundamental ideas of algebra and calculus as well as an appreciation of mathematics as an activity with its own intrinsic value, involving invention, intuition and exploration.

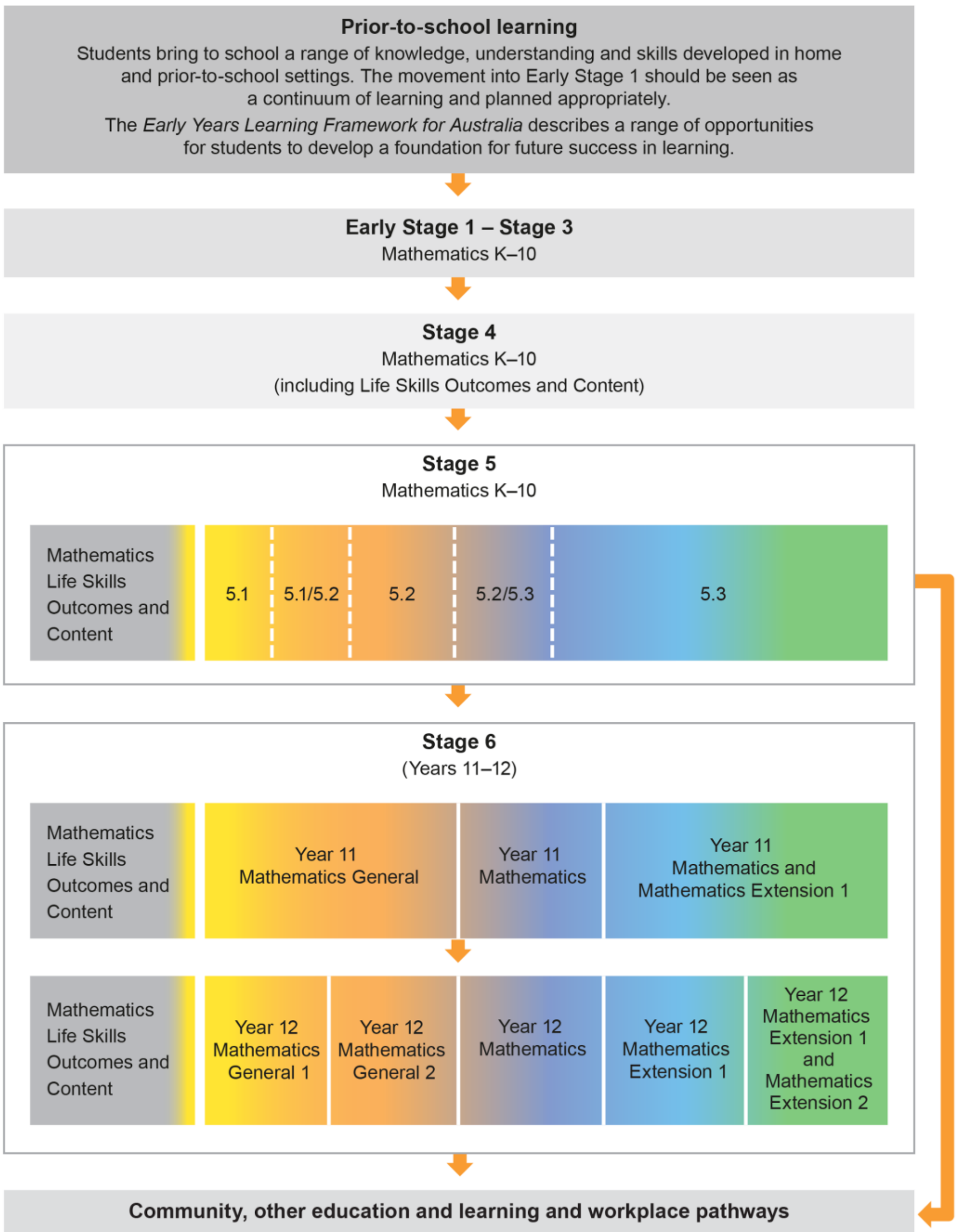
Mathematics Extension 2 provides a basis for a wide range of useful applications of mathematics as well as a strong foundation for further study of the subject.

THE PLACE OF THE MATHEMATICS EXTENSION 2 STAGE 6 DRAFT SYLLABUS IN THE K–12 CURRICULUM

for your information

NSW syllabuses include a diagram that illustrates how the syllabus relates to the learning pathways in K–12. This section places the Mathematics Extension 2 Stage 6 syllabus in the K–12 curriculum as a whole.

consult



AIM

for your information

In NSW syllabuses, the aim provides a succinct statement of the overall purpose of the syllabus. It indicates the general educational benefits for students from programs based on the syllabus.

The aim, objectives, outcomes and content of a syllabus are clearly linked and sequentially amplify details of the intention of the syllabus.

consult

The study of Mathematics Extension 2 in Stage 6 enables students to extend their knowledge and understanding of working mathematically, enhancing their skills to tackle difficult, unstructured problems, generalise, make connections and become fluent at communicating in a concise and systematic manner.

OBJECTIVES

for your information

In NSW syllabuses, objectives provide specific statements of the intention of a syllabus. They amplify the aim and provide direction to teachers on the teaching and learning process emerging from the syllabus. They define, in broad terms, the knowledge, understanding, skills, values and attitudes to be developed through study in the subject. They act as organisers for the intended outcomes.

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KNOWLEDGE, UNDERSTANDING AND SKILLS

Students:

- develop efficient strategies to solve complex problems using pattern recognition, generalisation and modelling techniques
- develop their knowledge, understanding and skills to solve complex and interconnected problems in the areas of functions, proof, vectors, calculus, statistical analysis and complex numbers
- develop their problem-solving and reasoning skills to create appropriate mathematical models in a variety of forms and apply these to difficult unstructured problems
- use mathematics as an effective means of communication and justification in complex situations

VALUES AND ATTITUDES

Students will value and appreciate:

- mathematics as an essential and relevant part of life, recognising that its development and use has been largely in response to human needs by societies all around the globe
- the importance of resilience and self-motivation in undertaking mathematical challenges

OUTCOMES

for your information

In NSW syllabuses, outcomes provide detail about what students are expected to achieve at the end of each Year in relation to the objectives. They indicate the knowledge, understanding and skills expected to be gained by most students as a result of effective teaching and learning. They are derived from the objectives of the syllabus.

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TABLE OF OBJECTIVES AND OUTCOMES – CONTINUUM OF LEARNING

Objectives Students:	**Outcomes** A student:
• develop efficient strategies to solve complex problems using pattern recognition, generalisation and modelling techniques	**MEX12-1** understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics
• develop their knowledge, understanding and skills to solve complex and interconnected problems in the areas of proof, vectors and mechanics, calculus and complex numbers	**MEX12-2** chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings
	MEX12-3 applies the ideas of vectors and mechanics to model and solve practical problems
	MEX12-4 solves first-order differential equations and applies further techniques of integration to unstructured problems
	MEX12-5 uses the relationship between algebraic and geometric representation of complex numbers and uses complex number techniques to solve problems
• develop their problem-solving and reasoning skills to create appropriate mathematical models in a variety of forms and apply these to difficult unstructured problems	**MEX12-6** applies various mathematical techniques and concepts to solve unstructured and multi-step problems

• use mathematics as an effective means of communication and justification in complex situations	**MEX12-7** communicates and justifies abstract ideas and relationships using appropriate notation and logical argument

COURSE STRUCTURE AND REQUIREMENTS

for your information

The following provides an outline of the Year 11 and Year 12 course structure and requirements for the *Mathematics Extension 2 Stage 6 Draft Syllabus* with indicative hours, arrangement of content, and an overview of course content.

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	Mathematics Extension 2	**Indicative hours**
Year 12 course (60 hours)	The course is organised into strands, with the strands divided into topics.	
	Proof	15
	Vectors and Mechanics	15
	Calculus	15
	Complex Numbers	15
	Modelling and applications are an integral part of each strand and merge strands together, enabling students to make connections and appreciate the use of mathematics and appropriate technology.	

For this course:

- the Year 12 Mathematics Extension 1 course should be taught prior or concurrently to this course
- 60 indicative hours are required to complete the course
- students should experience content in the course in familiar and routine situations as well as unfamiliar or contextual situations
- students should be provided with regular opportunities involving the integration of technology to enrich the learning experience

ASSESSMENT

ⓘ for your information

The key purpose of assessment is to gather valid and useful information about student learning and achievement. It is an essential component of the teaching and learning cycle. School-based assessment provides opportunities to measure student achievement of outcomes in a more diverse way than the HSC examination.

BOSTES continues to promote a standards-referenced approach to assessing and reporting student achievement. Assessment for, as and of learning are important to guide future teaching and learning opportunities and to give students ongoing feedback. These approaches are used individually or together, formally or informally, to gather evidence of student achievement against standards. Assessment provides teachers with the information needed to make judgements about students' achievement of outcomes.

Ongoing stakeholder feedback, analysis of BOSTES examination data and information gathered about assessment practices in schools has indicated that school-based and external assessment requirements require review and clarification. The HSC Reforms outline changes to school-based and HSC assessment practices to:

- make assessment more manageable for students, teachers and schools
- maintain rigorous standards
- strengthen opportunities for deeper learning
- provide opportunities for students to respond to unseen questions, and apply knowledge, understanding and skills to encourage in-depth analysis
- support teachers to make consistent judgements about student achievement.

Students with special education needs
Some students with special education needs will require adjustments to assessment practices in order to demonstrate what they know and can do in relation to syllabus outcomes and content. The type of adjustments and support will vary according to the particular needs of the student and the requirements of the assessment activity. Schools can make decisions to offer adjustments to coursework and school-based assessment.

Life Skills
Students undertaking Years 11–12 Life Skills courses will study selected outcomes and content. Assessment activities should provide opportunities for students to demonstrate achievement in relation to the outcomes, and to apply their knowledge, understanding and skills to a range of situations or environments.

The following general descriptions have been provided for consistency. Further advice about assessment, including in support materials, will provide greater detail.

Assessment for Learning	• enables teachers to use formal and informal assessment activities to gather evidence of how well students are learning • teachers provide feedback to students to improve their learning • evidence gathered can inform the directions for teaching and learning programs.
Assessment as Learning	• occurs when students use self-assessment, peer-assessment and formal and informal teacher feedback to monitor and reflect on their own learning, consolidate their understanding and work towards learning goals.
Assessment of Learning	• assists teachers to use evidence of student learning to assess student achievement against syllabus outcomes and standards at defined key points within a Year or Stage of learning.
Formal assessment	• tasks which students undertake as part of the internal assessment program, for example a written examination, research task, oral presentation, performance or other practical task • tasks appear in an assessment schedule and students are provided with sufficient written notification • evidence is gathered by teachers to report on student achievement in relation to syllabus outcomes and standards, and may also be used for grading or ranking purposes.
Informal assessment	• activities undertaken and anecdotal evidence gathered by the teacher throughout the learning process in a less prescribed manner, for example class discussion, questioning and observation • used as part of the ongoing teaching and learning process to gather evidence and provide feedback to students • can identify student strengths and areas for improvement.
Written examination	• a task undertaken individually, under formal supervised conditions to gather evidence about student achievement in relation to knowledge, understanding and skills at a point in time, for example a half-yearly, yearly or trial HSC examination • a task which may include one or more unseen questions or items, assessing a range of outcomes and content.

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Mathematics Extension 2 Draft Assessment Requirements
The draft guidelines for school-based assessment provide specific advice about the number of formal assessment tasks, course components and weightings, and the nature of task types to be administered in Year 12.

The components and weightings for Year 12 are mandatory.

Year 12

- There will be no more than 4 formal assessment tasks
- The maximum weighting for each formal assessment task is 40%
- One task may be a formal written examination, eg a trial HSC, with a maximum weighting of 25%
- One task must be an assignment or investigation-style task with a weighting of 20–30%.

Component	**Weighting %**
Knowledge, understanding and communication	50
Problem solving, reasoning and justification	50
	100

Mathematics Extension 2 Draft Examination Specifications
Option 1

Sections
Section – Written responses A variety of short answer questions to extended responses, with the possibility of a number of parts

Changes from current examination specifications
The objective response section will be removed. This approach provides opportunity for inclusion of cross-strand application questions, and modelling and problem solving questions to enable students to demonstrate deep understanding, conceptual knowledge, higher-order thinking and reasoning.

Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

Students undertaking Mathematics Extension 2 will also undertake the Mathematics Extension 1 examination.

Option 2

Sections
Section I – Objective responses
Section II – Written responses A variety of short answer questions to extended responses, with the possibility of a number of parts

Changes from current examination specifications
Questions or parts of questions may be drawn from a range of syllabus outcomes and content.

Students undertaking Mathematics Extension 2 will also undertake the Mathematics Extension 1 examination.

HSC examination specifications will be reviewed following finalisation of the syllabuses.

Updated assessment and reporting advice will be provided when syllabuses are released.

The Assessment Certification Examination website will be updated to align with the syllabus implementation timeline.

CONTENT

For Kindergarten to Year 12 courses of study and educational programs are based on the outcomes and content of syllabuses. The content describes in more detail how the outcomes are to be interpreted and used, and the intended learning appropriate for each Year. In considering the intended learning, teachers will make decisions about the emphasis to be given to particular areas of content, and any adjustments required based on the needs, interests and abilities of their students.

The knowledge, understanding and skills described in the outcomes and content provide a sound basis for students to successfully transition to their selected post-school pathway.

LEARNING ACROSS THE CURRICULUM

for your information

NSW syllabuses provide a context within which to develop core skills, knowledge and understanding considered essential for the acquisition of effective, higher-order thinking skills that underpin successful participation in further education, work and everyday life including problem-solving, collaboration, self-management, communication and information technology skills.

BOSTES has described learning across the curriculum areas that are to be included in syllabuses. In Stage 6 syllabuses, the identified areas will be embedded in the descriptions of content and identified by icons. Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the BOSTES *Statement of Equity Principles*, the *Melbourne Declaration on Educational Goals for Young Australians* (December 2008) and in the Australian Government's *Core Skills for Work Developmental Framework* (2013).

Knowledge, understanding, skills, values and attitudes derived from the learning across the curriculum areas will be included in BOSTES syllabuses, while ensuring that subject integrity is maintained.

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face.

The cross-curriculum priorities are:
- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

General capabilities encompass the knowledge, skills, attitudes and behaviours to assist students to live and work successfully in the 21st century.

The general capabilities are:
- Critical and creative thinking
- Ethical understanding
- Information and communication technology capability
- Intercultural understanding
- Literacy
- Numeracy
- Personal and social capability

BOSTES syllabuses include other areas identified as important learning for all students:

- Civics and citizenship
- Difference and diversity
- Work and enterprise

consult

Aboriginal and Torres Strait Islander histories and cultures

Through application and modelling across the strands of the syllabus, students can experience the relevance of Mathematics in Aboriginal and Torres Strait Islander histories and cultures. Opportunities are provided to connect Mathematics with Aboriginal and Torres Strait Islander peoples' cultural, linguistic and historical experiences. The narrative of the development of Mathematics and its integration with cultural development can be explored in the context of some topics. Through the evaluation of statistical data where appropriate, students can deepen their understanding of the lives of Aboriginal and Torres Strait Islander peoples.

When planning and programming content relating to Aboriginal and Torres Strait Islander histories and cultures teachers are encouraged to consider involving local Aboriginal communities and/or appropriate knowledge holders in determining suitable resources, or to use Aboriginal or Torres Strait Islander authored or endorsed publications.

Asia and Australia's engagement with Asia

Students have the opportunity to learn about the understandings and application of mathematics in Asia and the way mathematicians from Asia continue to contribute to the ongoing development of mathematics. By drawing on knowledge of and examples from the Asia region, such as calculation, money, art, architecture, design and travel, students can develop mathematical understanding in fields such as number, patterns, measurement, symmetry and statistics. Through the evaluation of statistical data, students can examine issues pertinent to the Asia region.

Sustainability

Mathematics provides a foundation for the exploration of issues of sustainability. Students have the opportunity to learn about the mathematics underlying topics in sustainability such as: energy use and how to reduce it; alternative energy with solar cells and wind turbines; climate change and mathematical modelling. Through measurement and the reasoned use of data students can measure and evaluate sustainability changes over time and develop a deeper appreciation of the world around them. Mathematical knowledge, understanding and skills are necessary to monitor and quantify both the impact of human activity on ecosystems and changes to conditions in the biosphere.

Critical and creative thinking

Critical and creative thinking are key to the development of mathematical understanding. Mathematical reasoning and logical thought are fundamental elements of critical and creative thinking. Students are encouraged to be critical thinkers when justifying their choice of a calculation strategy or identifying relevant questions during an investigation. They are encouraged to look for alternative ways to approach mathematical problems; for example, identifying when a problem is similar to a previous one, drawing diagrams or simplifying a problem to control some variables. Students are encouraged to be creative in their approach to solving new problems, combining the skills and knowledge they have acquired in their study of a number of different topics in a new context.

Ethical understanding

Mathematics makes a clear distinction between basic principles and the deductions made from them or their consequences in different circumstances. Students have opportunities to explore, develop and apply ethical understanding to mathematics in a range of contexts. Examples include: collecting, displaying and interpreting data; examining selective use of data by individuals and organisations; detecting and eliminating bias in the reporting of information; exploring the importance of fair comparison; and interrogating financial claims and sources.

Information and communication technology capability

Mathematics provides opportunities for students to develop ICT capacity when students investigate; create and communicate mathematical ideas and concepts using fast, automated, interactive and multimodal technologies. Students can use their ICT capability to perform calculations; draw graphs; collect, manage, analyse and interpret data; share and exchange information and ideas; and investigate and model concepts and relationships. Digital technologies, such as calculators, spreadsheets, dynamic geometry software, graphing software and computer algebra software, can engage students and promote understanding of key concepts.

Intercultural understanding

Students have opportunities to understand that mathematical expressions use universal symbols, while mathematical knowledge has its origin in many cultures. Students realise that proficiencies such as understanding, fluency, reasoning and problem-solving are not culture- or language-specific, but that mathematical reasoning and understanding can find different expression in different cultures and languages. The curriculum provides contexts for exploring mathematical problems from a range of cultural perspectives and within diverse cultural contexts. Students can apply mathematical thinking to identify and resolve issues related to living with diversity.

Literacy

Literacy is used throughout mathematics to understand and interpret word problems and instructions that contain the particular language features of mathematics. Students learn the vocabulary associated with mathematics, including synonyms, technical terminology, passive voice and common words with specific meanings in a mathematical context. Literacy is used to pose and answer questions, engage in mathematical problem-solving and to discuss, produce and explain solutions. There are opportunities for students to develop the ability to create and interpret a range of texts typical of mathematics, ranging from graphs to complex data displays.

Numeracy

Mathematics has a central role in the development of numeracy in a manner that is more explicit and foregrounded than is the case in other learning areas. It is related to a high proportion of the content. Consequently, this particular general capability is not tagged in the syllabus.

Numeracy involves drawing on knowledge of particular contexts and circumstances in deciding when to use mathematics, choosing the mathematics to use and critically evaluating its use. To be numerate is to use mathematics effectively to meet the general demands of life at home, in work, and for participation in community and civic life. It is therefore important that the mathematics curriculum provides the opportunity to apply mathematical understanding and skills in context, in other learning areas and in real-world contexts.

Personal and social capability

Students develop personal and social competence as they learn to understand and manage themselves, their relationships and their lives more effectively. Mathematics enhances the development of students' personal and social capabilities by providing opportunities for initiative taking, decision-making, communicating their processes and findings, and working independently and collaboratively in the mathematics classroom. Students have the opportunity to apply mathematical skills in a range of personal and social contexts. This may be through activities that relate learning to their own lives and communities, such as time management, budgeting and financial management, and understanding statistics in everyday contexts.

Civics and citizenship

Mathematics has an important role in civics and citizenship education because it has the potential to help us understand our society and our role in shaping it. The role of mathematics in society has expanded significantly in recent decades as almost all aspects of modern-day life are quantified. Through modelling reality with mathematics and then manipulating the mathematics in order to understand and/or predict reality, students have the opportunity to learn mathematical knowledge, skills and understanding that are essential for active participation in the world in which we live.

Difference and diversity

Students make sense of and construct mathematical ideas in different ways, drawing upon their own unique experiences in life and mathematics learning. By valuing students' diversity of ideas, teachers foster students' efficacy in learning mathematics.

Work and enterprise

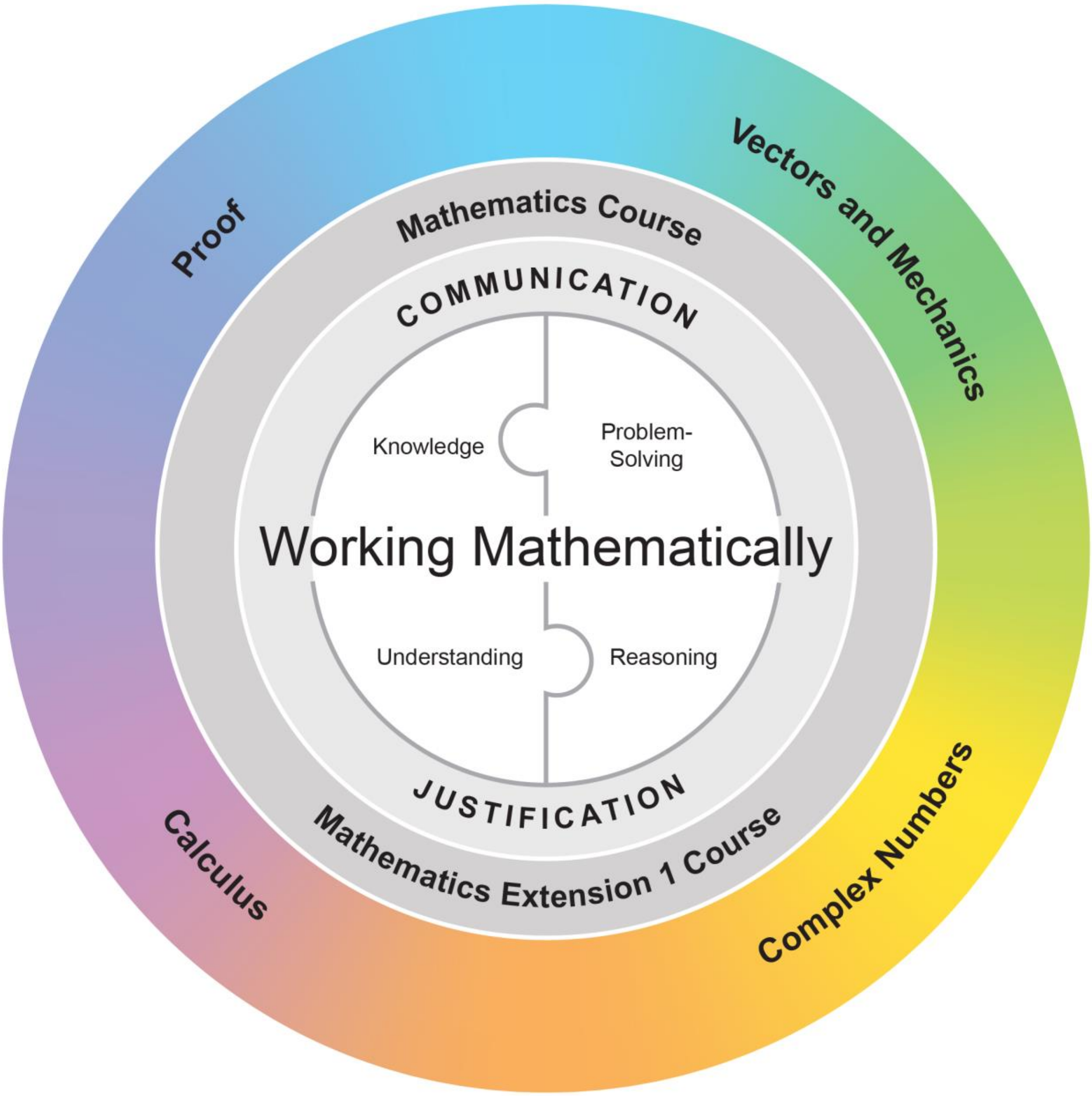
Students develop work and enterprise knowledge, understanding and skills through their study of mathematics in a work-related context. Students are encouraged to select and apply appropriate mathematical techniques and problem-solving strategies through work-related experiences in the financial mathematics and statistical analysis strands. This allows them to make informed financial decisions by selecting and analysing relevant information.

ORGANISATION OF CONTENT

for your information

The following provides a diagrammatic representation of the relationships between syllabus content.

consult



WORKING MATHEMATICALLY

Working Mathematically is integral to the learning process in mathematics. It provides students with the opportunity to engage in genuine mathematical activity and develop the skills to become flexible, critical and creative users of mathematics. In this syllabus, Working Mathematically is represented through two key components: *Knowledge, Understanding and Communication* and *Problem-Solving, Reasoning and Justification.* Together these form the focus of the syllabus, and the components of assessment.

Knowledge, Understanding and Communication

Students make connections between related concepts and progressively apply familiar mathematical concepts and experiences to develop new ideas. They develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily. Students communicate their chosen methods and efficiently calculated solutions, and develop an understanding of the relationship between the 'why' and the 'how' of mathematics. They represent concepts in different ways, identify commonalities and differences between aspects of content, communicate their thinking mathematically, and interpret mathematical information.

Problem-Solving, Reasoning and Justification

Students develop their ability to interpret, formulate, model and analyse identified problems and challenging situations. They describe, represent and explain mathematical situations, concepts, methods and solutions to problems, using appropriate language, terminology, tables, diagrams, graphs, symbols, notation and conventions. They apply mathematical reasoning when they explain their thinking, deduce and justify strategies used and conclusions reached, adapt the known to the unknown, transfer learning from one context to another, prove that something is true or false, and compare and contrast related ideas and explain choices. Their communication is powerful, logical, concise and precise.

Both components, and hence Working Mathematically, are evident across the range of syllabus strands, objectives and outcomes. Teachers extend students' level of proficiency in working mathematically by creating opportunities for development through the learning experiences that they design.

MATHEMATICS EXTENSION 2 YEAR 12 COURSE CONTENT

 consult

STRAND: PROOF

OUTCOMES

A student:

> chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings MEX12-2

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

STRAND FOCUS

Proof involves the communication and justification of mathematical argument in a concise and precise manner.

Knowledge of proof enables a level of communication that is accurate, concise and precise.

The study of proof is important in developing students’ ability to communicate, justify and critique mathematical arguments.

TOPICS

ME2-P1: The Nature of Proof
ME2-P2: Deductive Proof
ME2-P3: Proof by Mathematical Induction

PROOF

ME2-P1 THE NATURE OF PROOF

OUTCOMES

A student:

> chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings MEX12-2

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is to develop rigorous mathematical arguments and proofs specifically in the context of number.

Students develop an appreciation of the necessity for rigorous and robust methods to prove the validity of a variety of concepts related to number.

CONTENT

Students:

- use the language of proof, including but not limited to implication, converse, negation and contrapositive (ACMSM024)
  - use the symbols for implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), and equality (=), demonstrating a clear understanding of the difference between them (ACMSM026)
  - use the quantifiers 'for all' and 'there exists' (ACMSM027)
- use proof by contradiction (ACMSM025)
  - prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $log_2 5$ (ACMSM063)
- use examples and counter-examples (ACMSM028)
- prove simple results involving numbers, including but not limited to the test for divisibility by 3 or the sum of three consecutive even integers being divisible by 6 (ACMSM061)
- prove results involving inequalities and/or absolute values
  - prove linear or quadratic inequalities by use of the definition of $a > b$ for real a and b
  - prove linear or quadratic inequalities by using the property that squares of real numbers are non-negative
  - establish and use the relationship between the arithmetic mean and geometric mean for non-negative numbers
  - solve absolute value equations and inequations of the form $|ax \pm b| = k$ or $|ax \pm b| = |cx \pm d|$

PROOF

ME2-P2 DEDUCTIVE PROOF

OUTCOMES

A student:

> chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings MEX12-2

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is the use of deductive proof and its application in the context of circle geometry.

Students develop the ability to connect geometric representations and observations to formal proof arguments.

CONTENT

Students:

- Apply circle properties and prove theorems and converses in circle geometry using the following theorems: (ACMSM037)
  - Chords of equal length subtend equal angles at the centre (ACMSM033)
  - The perpendicular from the centre of a circle to a chord bisects the chord
  - Equal chords in equal circles are equidistant from the centres
  - The angle at the centre is twice the angle at the circumference subtended by the same arc (ACMSM030)
  - The angle in a semi-circle is a right angle (ACMSM029)
  - Angles in the same segment are equal (ACMSM031)
  - Opposite angles of a cyclic quadrilateral are supplementary (ACMSM032)
  - The exterior angle at the vertex of a cyclic quadrilateral is equal to the opposite interior angle
  - The tangent to a circle is perpendicular to the radius drawn to the point of contact
  - Tangents to a circle from an external point are equal
  - Two circles touch if they have a common tangent at the point of contact
  - When circles touch, the line of centres passes through the point of contact
  - The angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment (ACMSM034)
  - The products of the intercepts of two intersecting chords are equal (ACMSM035)
  - The square of the length of the tangent from an external point is equal to the product of the intercepts of the secant passing through this point (ACMSM036)
- Solve problems finding unknown angles and lengths and prove further results using the results listed above (ACMSM038)

PROOF

ME2-P3 PROOF BY MATHEMATICAL INDUCTION

OUTCOMES

A student:

> chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings MEX12-2

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is to use the technique of proof by mathematical induction to prove results in series, divisibility, inequality, calculus and geometry.

Students develop the use of formal mathematical language across various strands of mathematics to prove the validity of given situations involving inductive reasoning.

CONTENT

Students:

- understand and prove results using mathematical induction, including but not limited to results in series, divisibility tests, simple inequalities and results in algebra, calculus, probability and geometry
  - understand the nature of inductive proof including the 'initial statement' and inductive step (ACMSM064)
  - use sigma notation to abbreviate the sum of a series; for example, $\sum_{n=1}^{3}(10n+2)=12+22+32$.
  - prove results for sums, such as $1+4+9+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$ for any positive integer n (ACMSM065)
  - prove divisibility results, such as $3^{2n+4}-2^{2n}$ is divisible by 5 for any positive integer n (ACMSM066)
  - use mathematical induction to prove recursive formulae
  - recognise situations where proof by mathematical induction fails

STRAND: VECTORS AND MECHANICS

OUTCOMES

A student:

- > understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1
- > applies the ideas of vectors and mechanics to model and solve practical problems MEX12-3
- > applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6
- > communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

STRAND FOCUS

Vectors and Mechanics involves mathematical representation of quantity with magnitude and direction and its geometrical depiction.

Knowledge of vectors and mechanics enables understanding of behaviour of objects according to mathematical law.

Study of vectors and mechanics is important in developing students' understanding of space in two dimensions through the use of vectors and/or a calculus-based approach.

TOPICS

ME2-V1: Vectors
ME2-V2: Mechanics

VECTORS

ME2-V1 VECTORS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> applies the ideas of vectors and mechanics to model and solve practical problems MEX12-3

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is to explore representations of quantities with magnitude and direction in both two and three dimensions.

Students develop an understanding of connections between the behaviour of lines or quantities and their representation as vectors.

CONTENT

Students:

- use standard notations for vectors in both two and three dimensions, and are familiar with the terms 'position vector' and 'displacement vector' in abstract and practical contexts such as displacement and velocity (ACMSM010)
  - use cartesian coordinates in three-dimensional space (ACMSM103)
  - use column vector notation $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ to represent a vector (ACMSM014)
  - define and use unit vectors and the perpendicular unit vectors ***i***, ***j*** and ***k***, and express a vector in component form $x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ (ACMSM101, ACMSM015, ACMSM016)
  - use standard notation for vectors: $\overrightarrow{AB}$ and **a**
- perform addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometric terms (ACMSM012, ACMSM013)
  - examine and use addition and subtraction of vectors in component form (ACMSM017)
  - define and use multiplication by a scalar of a vector in component form (ACMSM018)
- calculate the magnitude of a vector and identify the magnitude of a displacement vector $\overrightarrow{AB}$ as being the distance between the points A and B (ACMSM011)
- calculate the scalar product of two vectors (in either two or three dimensions), and use the scalar product to determine the angle between two directions and to solve problems concerning perpendicularity of vectors (ACMSM019, ACMSM021)
  - apply the scalar product to vectors expressed in component form (ACMSM020)
- understand and use the vector form of a straight line $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$
  - determine a vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, both two and three dimensions (ACMSM105)
- use the cross product to determine a vector normal to a given plane (ACMSM107)

- determine whether two lines are parallel, intersect or are skew
- find the angle between two lines and the point of intersection of two lines when it exists
- use vectors and geometrical relationships to prove results in two or three dimensions, including but not limited to: the diagonals of a parallelogram meet at right angles if and only if it is a rhombus; and the midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM039, ACMSM040, ACMSM041, ACMSM102)

VECTORS AND MECHANICS

ME2-V2 MECHANICS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> applies the ideas of vectors and mechanics to model and solve practical problems MEX12-3

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is to model the mechanics of objects in a variety of situations with and without resistance using a calculus-based approach.

Students develop appreciation of mathematical representation and explanation of mechanics and their ability to construct models in situations involving resisted forces.

CONTENT

V2.1: Modelling motion without resistance

Students:

- define and use kinematics of motion in a straight line to solve problems in a variety of contexts including vertical motion under gravity
  - understand the difference between scalar and vector quantities and hence the relationship between distance, speed, displacement, velocity and acceleration
  - sketch and interpret graphs of displacement, velocity and acceleration with respect to time and hence prove the relationships between average rate of change, instantaneous rate of change, derivatives and integrals with respect to motion including but not limited to the formulae for motion with constant acceleration
- derive and use expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$ for acceleration (ACMSM136)
  - define the use of dotted notation for derivatives with respect to time
  - given $\ddot{x} = f(x)$ and initial conditions, $v^2 = g(x)$ describe the resultant motion
- determine and use equations of motion of a particle travelling in a straight line with both constant and variable acceleration (ACMSM114)
- determine force, acceleration, action and reaction (ACMSM133, ACMSM134, ACMSM135)
  - define and apply Newton's Laws of Motion to linear motion of bodies of constant mass moving under the action of constant forces, including but not limited to acceleration on an inclined plane or masses connected over a pulley system
- use vectors as representations of force to solve problems (ACMSM111)
  - relate forces to vectors and vector representations (in two dimensions only)
  - represent motion using vector component form
  - use vector addition and subtraction in solving problems involving resultants and components of forces
  - resolve vector systems to find perpendicular components of a given force

- define and use conservation of momentum in a variety of contexts, including the collision of two bodies (ACMSM133)
  - define and calculate momentum of a body under straight line motion and show an understanding of its vector nature

V2.2: Projectiles

Students:

- model the motion of a projectile as a particle moving with constant acceleration **M**
  - derive the horizontal and vertical equations of motion of a projectile
  - understand and explain the limitations of projectile models
- find the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached **M**
- derive and use the Cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown
- apply vector calculus to motion in a plane including projectiles (ACMSM113, ACMSM115) **M**
  - consider horizontal and vertical equations of motion and explain the limitations of the model
- use parametric equations of curves, and determine a 'corresponding' Cartesian equation (ACMSM104)
  - develop an understanding of parameter and differentiate parametric equations
- investigate historic sporting photographs that illustrate contentious umpire decisions before the advent of video umpiring. Using their understanding of motion, students must determine the validity of the umpire's decision justifying their reasons with appropriate mathematical argument, assumptions and justification **E**

V2.3: Simple Harmonic Motion

Students:

- solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion (ACMSM136) **M**
  - determine equations for displacement, velocity and acceleration given motion is periodic
  - solve problems relating to the period and range of periodic motion
- define and use Hooke's Law: $F = kx$ where F is force, x is the extension and k is a constant
  - explore the linkage between Hooke's Law and Simple Harmonic Motion from both a vector and calculus perspective
  - solve problems including but not limited to vertical strings and springs using both vector and calculus perspectives **M**

V2.4: Friction

Students:

- understand and use the principle of equilibrium of a particle to solve problems involving forces
  - identify the forces acting in a given situation, resolving where appropriate, and use the relationship between mass and weight
  - determine if a particle is in equilibrium by calculating the vector sum of the forces
- use a model of 'smooth' contact and understand the limitations of the model
- model and calculate with friction
  - represent the contact force between two rough surfaces by two components, the 'normal force' and the 'frictional force'
  - define and use the coefficient of static friction $\mu_s = \frac{f_{s\,max}}{n}$ where n is the normal force, and $f_{s\,max}$ is the maximum static friction
  - define and use the coefficient of kinetic friction $\mu_k = \frac{f_k}{n}$ where n is the normal force, and f_k is the constant kinetic friction
- apply motion-modelling techniques to practical situations **M**

– explore and solve situations involving other retarding forces such as rolling resistance, retarding rolling wheels, wind resistance or resistance to motion through any fluid

V2.5: Resisted Motion

Students:

- determine resisted motion along a horizontal line
  - derive an expression for velocity as a function of time or displacement where possible
  - derive an expression for displacement as a function of time where possible
- determine the model of motion of a particle moving upwards or downwards in a resisting medium and under the influence of gravity
  - derive an expression for velocity as a function of time or displacement where possible
  - derive an expression for displacement as a function of time where possible
  - determine the equation of motion of a particle, moving vertically upwards or downwards in a medium, with a resistance proportional to the first or second power of its speed
  - determine the terminal velocity of a falling particle from its equation of motion
- solve problems involving motion in practical situations using a calculus approach **M**

STRAND: CALCULUS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1
> solves first-order differential equations and applies further techniques of integration to unstructured problems MEX12-4
> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6
> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

STRAND FOCUS

Calculus involves broadening the range of techniques and strategies available in order to solve more complex problems involving differential equations and integration.

Knowledge of calculus enables an understanding of the concept of a derivative or the anti-derivative as a function which can be used to gain more information about the original function and using those functions to form equations to solve problems.

Study of calculus is important in developing students' knowledge and understanding of differentiating and integrating a variety of complex functions, and the ability to apply this to practical problems in a variety of situations.

TOPICS

ME2-C1: Advanced Calculus Skills

CALCULUS

ME2-C1 ADVANCED CALCULUS SKILLS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> solves first-order differential equations and applies further techniques of integration to unstructured problems MEX12-4

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is recognising and solving first-order differential equations and extending techniques to include integration techniques such as by parts.

Students develop the ability to recognise and apply this knowledge in a variety of contexts and multi-step problems.

CONTENT

C1.1: Differential Equations

Students:

- solve simple first-order differential equations (ACMSM130)
  - solve differential equations of the form $\frac{dy}{dx} = f(x)$
  - solve differential equations of the form $\frac{dy}{dx} = g(y)$
  - solve differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables
- formulate differential equations including but not limited to the logistic equation that will arise in, for example, chemistry, biology and economics in situations where rates are involved (ACMSM132) **M**

C1.2: Integration Techniques

Students:

- integrate rational functions by completing the square in a quadratic denominator
- use partial fractions to integrate functions of the form $\frac{f(x)}{g(x)}$ where *f(x)* and *g(x)* are polynomials and *g(x)* is expressed in simple linear or quadratic factors (ACMSM122)
- find and evaluate integrals using the method of integration by parts (ACMSM123)
  - develop the method for integration by parts: $\int uv'dx = uv - \int vu'dx$ where u and v are both functions of x
- apply the techniques from section C1.1 and C1.2 to practical applications **M**

STRAND: COMPLEX NUMBERS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> uses the relationship between algebraic and geometric representation of complex numbers and uses complex number techniques to solve problems MEX12-5

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

STRAND FOCUS

Complex Numbers involves investigating and extending understanding of the real number system to include complex numbers. The use of complex numbers is integral to life and modern-day technology.

Knowledge of complex numbers enables an understanding of how different forms of number affect calculations and the solution of problems.

A study of complex numbers is important in developing students’ understanding of the interconnectedness of mathematics and the real world.

TOPICS

ME2-N1: Introduction to Complex Numbers
ME2-N2: Using Complex Numbers

COMPLEX NUMBERS

ME2-N1 INTRODUCTION TO COMPLEX NUMBERS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> uses the relationship between algebraic and geometric representation of complex numbers and uses complex number techniques to solve problems MEX12-5

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is the development of the concept of complex numbers, using different representations of complex numbers and solving associated problems.

Students develop a suite of tools to both represent and operate with complex numbers.

CONTENT

Students:

- use the complex number system and its subsets
  - develop an understanding of the classification structure of numbers, their associated properties, symbols and representations
  - define the imaginary number i as a root of the equation $x^2 = -1$ (ACMSM067)
- represent and use complex numbers in Cartesian form (rectangular form)
  - use complex numbers in the form $z = a + bi$ where a and b are the real $Re(z)$ and imaginary $Im(z)$ parts (ACMSM068)
  - identify conditions for $a + bi$ and $c + di$ to be equal
  - define, find and use complex conjugates (ACMSM069)
  - find the reciprocal of complex number of the form $z = a + bi$
  - perform complex number addition, subtraction, multiplication and division (ACMSM070)
  - divide a complex number $a + bi$ by another complex number $c + di$
  - find the square roots of a complex number $a + bi$
  - prove that there are always two roots of a non-zero complex number
- represent and use complex numbers in the Argand plane (Argand diagram)
  - consider, use and represent complex numbers as points in a plane with real and imaginary parts as cartesian coordinates (ACMSM071)
  - recognise the geometrical relationship between the points representing a complex number z and the points representing $\bar{z}$ and cz (ACMSM073)
  - examine and use the effect of multiplication of a complex number, z, by i^n and rotational movements (ACMSM085)
  - represent complex numbers as vectors on an Argand diagram
  - examine and use addition and subtraction of complex numbers as vectors in the Argand plane (ACMSM084)

  - explore, describe and determine geometrical relationships using vector representations of complex numbers, for example describing the vector representing $z = z_1 + z_2$ as corresponding to the diagonal of a parallelogram with vectors representing z_1 and z_2 as adjacent sides (ACMSM072)
- represent and use complex numbers in polar form
  - define and use the modulus $|z|$ of a complex number z and the argument $arg(z)$ of a non-zero complex number z (ACMSM080)
  - define and use the principal argument $Arg(z)$ of a non-zero complex number z.
  - prove and use the basic identities involving modulus and argument: (ACMSM080)
    $|z_1 z_2| = |z_1|.|z_2|$ and $arg\ (z_1 z_2) = arg\ z_1 + arg\ z_2$
    $|\frac{z_1}{z_2}| = \frac{|z_1|}{|z_2|}$ and $arg\ (\frac{z_1}{z_2}) = arg\ z_1 - arg\ z_2$
    $|z^n| = |z|^n$ and $arg\ (z^n) = n\ arg\ z$
    $|\frac{1}{z^n}| = \frac{1}{|z|^n}$ and $arg\ \left(\frac{1}{z^n}\right) = -n\ arg\ z$
    $\overline{z_1} + \overline{z_2} = \overline{z_1 + z_2}$
    $\overline{z_1}\ \overline{z_2} = \overline{z_1\ z_2}$
  - convert complex numbers between cartesian, polar and geometrical representations (ACMSM081)
  - use multiplication, division and powers of complex numbers in polar and geometric representations (ACMSM082)
- represent and use complex numbers in exponential form
  - use Euler's Formula, $e^{ix} = cos\ x + i\ sin\ x$, to link polar form and exponential form
  - use powers of complex numbers in exponential form

COMPLEX NUMBERS

ME2-N2 USING COMPLEX NUMBERS

OUTCOMES

A student:

> understands and uses different representations of numbers and functions to find solutions to problems in a variety of contexts and areas of mathematics MEX12-1

> uses the relationship between algebraic and geometric representation of complex numbers and uses complex number techniques to solve problems MEX12-5

> applies various mathematical techniques and concepts to solve unstructured and multi-step problems MEX12-6

> communicates and justifies abstract ideas and relationships using appropriate notation and logical argument MEX12-7

TOPIC FOCUS

The principal focus of this topic is to apply complex number knowledge to situations involving trigonometric identities, powers and vector representations in a complex number plane.

Students develop appreciation of the interconnectedness of complex numbers across various strands of mathematics and their application in real life.

CONTENT

N2.1: Solving Equations with Complex Numbers

Students:

- use De Moivre's theorem with trigonometric identities and powers of complex numbers
  - prove De Moivre's theorem for integral powers using Euler's Formula (ACMSM083)
  - solve power equations (ACMSM082)
- solve quadratic equations $ax^2 + bx + c = 0$ where a, b, c are complex numbers
  - use the general solution of real quadratic equations (ACMSM074)
  - determine complex conjugate solutions of real quadratic equations (ACMSM075)
- solve complex number equations $az^2 + bz + c = 0$ where z is a complex number
- determine conjugate roots for polynomials with real coefficients (ACMSM090)

N2.2: Geometrical Implications of Complex Numbers

Students:

- examine and use addition of complex number as vector addition in the Argand plane
- examine and use multiplication as a geometric interpretation such as rotation and scaling in the Argand plane (ACMSM085)
- determine and examine the $n\ th$ roots of unity (degree ≤ 3) and their location on the unit circle (ACMSM087)
- determine and examine the $n\ th$ roots of complex numbers (degree ≤ 3) and their location in the Argand plane (ACMSM088)
- identify subsets of the Argand plane determined by relations such as $|z - 3i| \leq 4$ $\frac{\pi}{4} \leq Arg(z) \leq \frac{3\pi}{4}$, $Re(z) > Im(z)$ and $|z - 1| = 2|z - i|$ (ACMSM086)

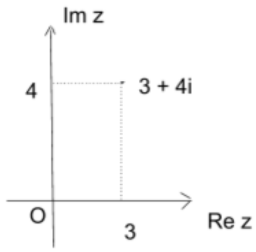
- explore different visual representations of complex numbers (fractals) and their behaviour, including but not limited to prisoner and escape sets, self-similarity and the Mandelbrot set, defined by a family of complex quadratic polynomials $P_C: C \to C$ given by $P_C: z \to z^2 + c$ **E**
- research real-life applications of complex numbers such as use in electric circuits and design principles **E**

GLOSSARY

for your information

The glossary explains terms that will assist teachers in the interpretation of the subject. The glossary will be based on the NSW Mathematics K–10 glossary and the Australian curriculum senior secondary years Specialist Mathematics glossary.

consult

Glossary term	Definition
alternate segment	A segment of a circle is the region that lies between a chord of a circle and its associated arc. When a single chord is drawn in a circle, the circle is divided into two segments. The smaller segment is known as the minor segment while the larger is known as the major segment. When you consider one segment, the other segment is known as the 'alternate segment'.
Argand plane	The Argand plane is a geometric representation of the complex numbers established by the real axis and the orthogonal imaginary axis. The Argand plane is also referred to as the Argand diagram. 
argument (of a complex number)	When a complex number, z, is represented by a point P in the complex plane then the argument of z, denoted $arg\ z$, is the angle θ that OP(where O denotes the origin) makes with the positive real axis O_x , with the angle measured anticlockwise from O_x. $Arg\ z$ is defined as the principal value of the argument, and is restricted to the interval $(-\pi, \pi]$.
Cartesian form (of a complex number)	The Cartesian form of a complex number is $a + bi$, where a and b are real numbers and i is the imaginary number.
column vector notation	A vector can be represented in ordered pair notation as, for example, $(4,5)$. In column vector notation, the same vector would be notated as $\frac{4}{5}$.

Glossary term	Definition
complex conjugate	For any complex number $z = a + bi$, its conjugate is $\bar{z} = a - bi$ where a and b are real numbers and i is the imaginary number.
complex number	A complex number is a number that can be expressed in the form $a + bi$, where a and b are real numbers and i is the imaginary number which satisfies the equation $i^2 = -1$.
contrapositive	The contrapositive of the statement 'If Pthen Q' is 'If not Q then not P'. The contrapositive of a true statement is also true. Example: **Statement:** If $x = 2$ then $x^2 = 4$. **Contrapositive:** If $x^2 \neq 4$ then $x \neq 2$.
converse of a statement	The converse of a statement 'If P then Q' is 'If Q then P' Symbolically the converse of $P \Rightarrow Q$ is: $Q \Rightarrow P$ or $P \Leftarrow Q$ The converse of a true statement need not be true. Examples: **Statement:** If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other. **Converse statement:** If the diagonals of a quadrilateral are of equal length and bisect each other then the quadrilateral is a rectangle. (In this case the converse is true.) **Statement:** If $x = 2$ then $x^2 = 4$. **Converse statement:** If $x^2 = 4$ then $x = 2$. (In this case the converse is false.)
counterexample	A counterexample is an example that demonstrates that a statement is not true. Examples: **Statement:** If $x^2 = 4$ then $x = 2$. **Counterexample:** $x = -2$ provides a counterexample.
cyclic quadrilateral	A cyclic quadrilateral is a quadrilateral whose vertices lie on a circle.
De Moivre's theorem	For all integers n, $[r(cos\ \theta\ +\ i\ sin\ \theta)]^n = r^n(cos\ n\theta\ +\ i\ sin\ n\theta)$. This can also be written as: $[rcis\theta]^n = r^n(cisn\theta)$
displacement vector	A displacement vector describes the displacement from one point to another.
equidistant	A point is said to be equidistant from a set of objects if the distances between that point and each object in the set are equal.

Glossary term	Definition
Euler's formula	Euler's formula for complex numbers establishes the fundamental relationship between trigonometric functions and the complex exponential function. It states that for any real number x: $e^{ix} = cos\ x\ +\ i\ sin\ x$
exponential form (of a complex number)	The exponential form of a complex number is $re^{i\theta}$ where r is the modulus of the complex number, and θ is the argument expressed in radians.
imaginary number	An imaginary number is any number that can be expressed as bi, where b is any real number, and i is the imaginary number that satisfies the equation $i^2 = -1$.
implication	To say that P implies Q is to say that the truth of Q may be deduced from the truth of P. In shorthand it can be written as 'If P then Q' and in notation form as $P \Rightarrow Q$. Examples: ● If a quadrilateral is a rectangle, then the diagonals are of equal length and they bisect each other. ● If $x = 2$ then $x^2 = 4$. ● If an animal is a kangaroo, then it is a marsupial. ● If a quadrilateral is cyclic then the opposite angles are supplementary.
integration by parts	Integration by parts is a technique for performing indefinite or definite integration given by: $\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx} dx$
mathematical induction	Let there be associated with each positive integer n, a proposition $P(n)$ 1. If $P(1)$ is true and 2. If the truth of $P(k)$ implies the truth of $P(k+1)$ for every positive integer k, then $P(n)$ is true for all positive integers n.
modulus (of a complex number)	If z is a complex number and $z = a + bi$ then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z denoted by $\lvert z\rvert$, and $\lvert z\rvert = \sqrt{a^2 + b^2}$.
negation	If P is a statement then the statement 'not P', denoted by $\neg P$ is the negation of P. For example, if P is the statement 'It is snowing', then $\neg P$is the statement 'It is not snowing'.
polar form	The polar form of a complex number is $r(cos\ \theta + isin\ \theta)$ where r is the modulus of the complex number, and θ is the argument expressed in radians.

Glossary term	Definition
position vector	A position vector represents the position of a point P in space from a central point, usually from the origin.
principal argument	When a complex number is represented by a point P in the complex plane then the argument of z, denoted $arg\ z$, is the angle θ that OP(where O denotes the origin) makes with the positive real axis O_x , with the angle measured anticlockwise from O_x. $Arg\ z$ is defined as the principal value of the argument, and is restricted to the interval $(-\pi, \pi]$.
proof by contradiction	Assume the opposite (negation) of what you are trying to prove. Then proceed through a logical chain of argument until you reach a demonstrably false conclusion. Since all the reasoning is correct and a false conclusion has been reached, the only thing that could be wrong is the initial assumption. Therefore, the original statement is true. For example: the result ‘ $\sqrt{2}$ is irrational’ can be proved in this way by obtaining a contradiction from the assumption that $\sqrt{2}$ is rational.
rational number	A rational number is a number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
real number	A real number is either rational or irrational. The set of real numbers consists of the set of all rational and irrational numbers.
recurrence relations	A recurrence relation is described by a recursive formula.
recursive formula	A recursive formula defines a sequence. Once one or more initial terms are given, each further term of the sequence is defined as a function of preceding terms.
resisted motion	Resisted motion is motion that encounters resisting forces, for example friction and air resistance.
roots of unity	The complex number z is an nth root of unity if $z^n = 1$. Such a z is given by: $cos\ \frac{2k\pi}{n} + i\ sin\ \frac{2k\pi}{n}$ where $k = 0,1,2,\ldots, n-1$. The points in the complex plane representing the roots of unity lie on the unit circle.

Glossary term	Definition
scalar product	Also known as the 'dot product', the scalar product of vectors a and b is notated as $a.b$ If $a = \binom{a_1}{a_2}$ and $b = \binom{b_1}{b_2}$ then the scalar product $a.b$ is the real number $a_1b_1 + a_2b_2$. When expressed in i, j notation, $a = a_1\,i + a_2\,j$ and $b = b_1\,i + b_2\,j$ then $a.b = a_1b_1 + a_2b_2$
secant	A straight line passing through two points of a circle is called a secant.
sigma notation	The summation symbol Σ is used to describe the summation of a sequence (to give a series). $\sum_{k=m}^{n} a_k = a_m + a_{m+1} + \ldots + a_{n-1} + a_n$ For example, $\sum_{k=3}^{6} k^2 = 3^2 + 4^2 + 5^2 + 6^2 = 86$
subtend	The angle that an interval or arc with endpoints A and B subtends at a point P is angle APB. For example, the angle subtended by a diameter of a circle at any point on its circumference is a right angle.
supplementary	Two angles are supplementary if they add up to 180^o.
tangent	The tangent line (or tangent) to a curve at a given point P can be described intuitively as the straight line that "just touches" the curve at that point.
unit vector	A unit vector has direction, and its magnitude is 1. The most common unit vectors used are ***i*** (a vector of unit 1 in the x-direction) and ***j*** (a vector of unit 1 in the y-direction).
vector	A vector is a quantity that has magnitude and direction.